In other words, the most economical number of electrotypes to use for this job is 24. Of course only the nearest integral value of x is used.

The general problem is stated thus:

$$C = \frac{PR}{S(1+x)} + E \cdot x,$$

$$\frac{dC}{dx} = -\frac{PR}{S(1+x)^2} + E = 0,$$

for a minimum.

From which

$$x = \sqrt{\frac{PR}{ES}} - 1.$$

This gives a formula involving only the arithmetical work of finding the square root to determine the number of electrotypes needed in any given case, and one of easy application by any practical printer.

RECENT PUBLICATIONS.

REVIEWS.

Differential Equations. By H. Bateman. London, Longmans, Green and Co., 1918. 8vo. 11 + 306 pp. Price 16 shillings.

The study of elementary methods of integrating differential equations is one which is taken up in many American colleges in a course following the integral calculus, or sometimes as a part of that course. When properly taught, it is a subject admirably adapted to developing in the student a skillful technique in using his calculus, a thing which he will find most helpful in his later work. Many students coming from calculus are woefully weak in many parts of the work which they have studied and "passed," so if such students are to go on to differential equations the beginning, at least, must be easy. They will then have some chance to develop and show their real ability. However, the manipulative side of the study must not be over-emphasized, for the extensive theoretical parts must be suitably developed. Moreover, there is far more opportunity for geometrical discussions than is generally given.

Since the time of Boole many text books on elementary differential equations have appeared in England and America. The general plan of all these books has, however, been much the same. Differential equations were classified into certain "standard forms," and, after having discussed the methods to be used in integrating these type forms, problems were given falling more or less closely under them. The number of real "clothed problems" was usually small. The book here under review is entirely different both in arrangement and content from

the earlier books. This is especially true in the first four chapters. To quote from the preface: "The order in which the material has been arranged is slightly different from that which is usually adopted. Instead of beginning with the standard forms of equations which can be solved very easily, I have devoted the second chapter to integrating factors, and the third to the method of transformations."

When one comes to examine those changes in arrangement they appear to be considerable in the early chapters. The book has eleven chapters as follows: I. Differential equations and their solutions. II. Integrating factors. III. Transformations. IV. Geometrical applications. V. Differential equations, with particular solutions of a specified type. VI. Partial differential equations. VII. Total differential equations. VIII. Partial differential equations of the second order. IX. Integration in series. X. The solution of linear differential equations by means of definite integrals. XI. The mechanical integration of differential equations. At the end there are eight pages of miscellaneous examples, and an index.

With this list of chapter headings before him, let the reader ask himself where he would look for some topic with which he is familiar. Take, for instance, the general linear equation with constant coefficients. The index does not help, but a process of elimination leads one to look in the table of contents under Chapters II and III. Search there fails to give an exact reference, but linear second order equations are treated on p. 28 of Chapter II. Eight pages later we find the general linear equation with constant coefficients. However, the author does not treat it by means of an integrating factor, but by the symbolic method.

The question of the arrangement of the subject matter is of course extremely important. If a new arrangement is better than the old one, such comments as the foregoing would mean little. The author's plan in the first four chapters seems to be to classify his equations according to the methods used in discussing them, but he abandons this plan in the later chapters. The older plan classified differential equations according to their form, and integrated them by any available method. This older plan has one great advantage over the author's method, for it gives the student a classification of the equations themselves. When an ordinary differential equation is proposed for solution, the plan of attack is to discover which one of many methods is to be used. For this purpose some method of classification must be followed, and the most obvious classification is first according to order, and then by the form of the equation. If then the text book follows this same classification the student will be more ready to use it in attacking an unsolved equation. On the other hand the author's plan has the obvious advantage arising from the application of somewhat similar methods to equations of widely different forms. It leads to some curious results. Thus we find discontinuous solutions of linear equations with constant coefficients and a discussion of the general theory of a simultaneous system of n linear equations coming before such simple matters as the integrating factor of an equation of the

first order and degree, and the solution of the homogeneous equation of the first order. Such reversals of difficulty are frequent throughout the book. If Chapters II and III were interchanged they would be easier for the student. However, the author has small regard for the student's ease. Formidable computations occur frequently. Perhaps this conduces to that manipulative skill which the reviewer advocated in the beginning.

The book contains a large amount of valuable material, both in the text and in the problems. In the text, under ordinary differential equations, the discussion of linear equations and simultaneous systems of linear equations is especially complete. The author makes many interesting applications of these results to the discussion of problems in electricity, radioactivity, and dynamical systems. Many of these involve far more extensive investigations of the supplementary conditions than are given in the older texts, and some of them involve discontinuous solutions. Among the problems for the student to solve are many more of the same kind.

In Chapter V we find discussions of Euler's equation, the hypergeometric equation, Bessel's and Legendre's equations. These equations are further treated in Chapters IX and X. In Chapter IX. are also given Cauchy's existence theory for the ordinary first order equation, the method of successive approximations for that equation and Runge's method of approximate solution; certain convergence proofs for the linear equation of the second order, and for a simultaneous system of two first order equations. Here also we find a discussion of some of the expansion problems of physical mathematics.

In Chapter X we find a satisfactory account of the method of solving linear differential equations by means of definite integrals, and the Green's function of a linear partial differential equation with assigned boundary conditions.

Let us now return to Chapter VI on Partial Differential Equations of the first order. The whole treatment of this chapter is unsatisfactory. It strikes the reader as being one mass of formulas (*e.g.*, pp. 132, 133). That this is unnecessary is seen by consulting Goursat.[1] The author leaves almost untouched the whole of the beautiful geometrical relations between the various integral surfaces and the characteristic strips, as developed, for example, by Lie.[2] It is true that the author derives the differential equations which define the characteristic strips, but he scarcely uses the result. The fact that the integral surface is in general determined by making it pass through a given curve, but fails to be determined when and only when that curve is a *characteristic* curve, is unnoticed.

Chapter VII deals with total differential equations. In transforming variables in such an equation the author uses a symbolic method and introduces some results of his own.

Chapter VIII opens with a discussion of the homogeneous linear partial differential equation of the second order. This is followed by extensive applica-

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[1] *Equations aux dérivées partielles du premier ordre*, Paris, 1891, pp. 90–92 and pp. 102–107.

[2] *Berührungstransformationen*, Leipzig, 1896.

tions to important physical problems, including wave propagation, Maxwell's equations, theory of electrons, Laplace's equation and harmonic equations.

The last chapter describes various mechanical contrivances for solving certain differential equations of special forms. Several of these are due to E. Pascal, who has devised a number of instruments for this purpose.

Throughout the book there are many typographical errors. Most of them are easily corrected by the reader, but at least one of them has caused trouble to an unwary instructor.

The reviewer feels that the book will be difficult reading for a student beginning the study of differential equations, but that can be determined only by trying it out with a class. The book will surely prove to be a valuable addition to the library of the worker in mathematical physics.

CHARLES L. BOUTON.

HARVARD UNIVERSITY,
June, 1920.

The Theory of Determinants in the Historical Order of Development. By THOMAS MUIR. Volume 3, the period 1861 to 1880. London, Macmillan, 1920. 8vo. 26 + 503 pp. Price 35 shillings.

The first edition of the first volume of this work was reprinted in book form, in 1890, from the *Proceedings of the Royal Society of Edinburgh*; a second edition, with over 200 pages of additional material, appeared in 1906, and covered the history of general and special determinants up to 1841. The second volume (1911) made a similar survey for the period 1841 to 1860. The third volume under review covers an additional twenty year period. In June, 1918, the manuscript of a fourth volume bringing the record up to the end of the nineteenth century was nearly complete. Mathematicians must ever be grateful to Sir Thomas for his monumental work which is designed to contain a complete record of published results in connection with the theory of determinants.

The material is admirably arranged and indexed so that it is possible readily to trace the contributions to the theory of any individual, or the chronological development of any special type of determinants. For example chapter 14 in volume 2 and chapter 15 in volume 3 contain the history of circulants from the first paper of Catalan in 1846 to the last of Gegenbaur in 1880; skew determinants may be traced in a similar way in the fourteenth, ninth and tenth chapters of volumes 1, 2 and 3 respectively. Although a three page chapter is devoted to "cubic and n-dimensional determinants up to 1880" and various titles are listed, practically as in the article of 1900 by Professors Hedrick and Cairns,[1] the contents of the papers are not analyzed as in the other chapters because the work in question is a survey of determinants as ordinarily defined, and not of their generalizations.

The titles of the chapters are as follows—I: "Determinants in general, from 1860 to 1880," 1–82; II: "Determinants and linear equations, from 1861 to 1878," 83–93; III: "Axisymmetric

[1] "On three dimensional determinants," *Annals of Mathematics*, second series, vol. 1, pp. 49–67.